Mining Data Streams (Part 1)

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New Topic: Infinite Data

High dim. data
- Locality sensitive hashing
- Clustering
- Dimensionality reduction

Graph data
- PageRank, SimRank
- Community Detection
- Spam Detection

Infinite data
- Filtering data streams
- Queries on streams
- Web advertising

Machine learning
- SVM
- Decision Trees
- Perceptron, kNN

Apps
- Recommender systems
- Association Rules
- Duplicate document detection

Data Streams

- In many data mining situations, we do not know the entire data set in advance

- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates

- We can think of the data as **infinite** and **non-stationary** (the distribution changes over time)
Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)

- **We call elements of the stream tuples**

The system cannot store the entire stream accessibly

**Q:** How do you make critical calculations about the stream using a limited amount of (secondary) memory?
Stochastic Gradient Descent (SGD) is an example of a stream algorithm.

In Machine Learning we call this: **Online Learning**
- Allows for modeling problems where we have a continuous stream of data
- We want an algorithm to learn from it and slowly adapt to the changes in data

**Idea: Do slow updates to the model**
- SGD (SVM, Perceptron) makes small updates
- **So:** First train the classifier on training data.
- **Then:** For every example from the stream, we slightly update the model (using small learning rate)
General Stream Processing Model

Streams Entering. Each is stream is composed of elements/tuples

1, 5, 2, 7, 0, 9, 3
a, r, v, t, y, h, b
0, 0, 1, 0, 1, 1, 0

Ad-Hoc Queries

Processor

Standing Queries

Output

Limited Working Storage

Archival Storage
Types of queries one wants on answer on a data stream: (we’ll do these today)

- **Sampling data from a stream**
  - Construct a random sample

- **Queries over sliding windows**
  - Number of items of type $x$ in the last $k$ elements of the stream
Types of queries one wants on answer on a data stream: (we’ll do these next time)

- Filtering a data stream
  - Select elements with property \( x \) from the stream

- Counting distinct elements
  - Number of distinct elements in the last \( k \) elements of the stream

- Estimating moments
  - Estimate avg./std. dev. of last \( k \) elements

- Finding frequent elements
Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday

- **Mining click streams**
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour

- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook
Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller

- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies

- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks
Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger
Since we cannot store the entire stream, one obvious approach is to store a sample.

Two different problems:

1. Sample a fixed proportion of elements in the stream (say 1 in 10)

2. Maintain a random sample of fixed size over a potentially infinite stream
   - At any “time” $k$ we would like a random sample of $s$ elements
   - What is the property of the sample we want to maintain?
     For all time steps $k$, each of $k$ elements seen so far has equal prob. of being sampled.
Problem 1: Sampling fixed proportion

Scenario: Search engine query stream

- Stream of tuples: (user, query, time)
- Answer questions such as: How often did a user run the same query in a single days
- Have space to store \( \frac{1}{10} \)th of query stream

Naïve solution:

- Generate a random integer in \([0..9]\) for each query
- Store the query if the integer is 0, otherwise discard
Simple question: What fraction of queries by an average search engine user are duplicates?

- Suppose each user issues $x$ queries once and $d$ queries twice (total of $x+2d$ queries)
  - Correct answer: $d/(x+d)$

Proposed solution: We keep 10% of the queries

- Sample will contain $x/10$ of the singleton queries and $2d/10$ of the duplicate queries at least once
- But only $d/100$ pairs of duplicates
  - $d/100 = 1/10 \cdot 1/10 \cdot d$
  - Of $d$ “duplicates” $18d/100$ appear exactly once
    - $18d/100 = ((1/10 \cdot 9/10)+(9/10 \cdot 1/10)) \cdot d$

So the sample-based answer is $\frac{d}{x + \frac{d}{10} + \frac{18d}{100}} = \frac{d}{10x+19d}$
Solution: Sample Users

Solution:

- Pick $\frac{1}{10}$th of users and take all their searches in the sample.

- Use a hash function that hashes the user name or user id uniformly into 10 buckets.
Stream of tuples with keys:
- Key is some subset of each tuple’s components
  - e.g., tuple is (user, search, time); key is user
- Choice of key depends on application

To get a sample of $a/b$ fraction of the stream:
- Hash each tuple’s key uniformly into $b$ buckets
- Pick the tuple if its hash value is at most $a$

Hash table with $b$ buckets, pick the tuple if its hash value is at most $a$.
How to generate a 30% sample?
Hash into $b=10$ buckets, take the tuple if it hashes to one of the first 3 buckets
Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size
Problem 2: Fixed-size sample

Suppose we need to maintain a random sample $S$ of size exactly $s$ tuples

- E.g., main memory size constraint

Why? Don’t know length of stream in advance

Suppose at time $n$ we have seen $n$ items

- Each item is in the sample $S$ with equal prob. $s/n$

How to think about the problem: say $s = 2$

Stream: $[a, x, c, y, z, k, c, d, e, g,...$

At $n = 5$, each of the first 5 tuples is included in the sample $S$ with equal prob.
At $n = 7$, each of the first 7 tuples is included in the sample $S$ with equal prob.

Impractical solution would be to store all the $n$ tuples seen so far and out of them pick $s$ at random
**Algorithm (a.k.a. Reservoir Sampling)**

- Store all the first $s$ elements of the stream to $S$
- Suppose we have seen $n-1$ elements, and now the $n^{th}$ element arrives ($n > s$)
  - With probability $s/n$, keep the $n^{th}$ element, else discard it
  - If we picked the $n^{th}$ element, then it replaces one of the $s$ elements in the sample $S$, picked uniformly at random

**Claim:** This algorithm maintains a sample $S$ with the desired property:

- After $n$ elements, the sample contains each element seen so far with probability $s/n$
We prove this by induction:

- Assume that after $n$ elements, the sample contains each element seen so far with probability $s/n$.
- We need to show that after seeing element $n+1$, the sample maintains the property.
  - Sample contains each element seen so far with probability $s/(n+1)$.

Base case:
- After we see $n=s$ elements the sample $S$ has the desired property.
  - Each out of $n=s$ elements is in the sample with probability $s/s = 1$. 

Proof: By Induction

- **Inductive hypothesis:** After $n$ elements, the sample $S$ contains each element seen so far with prob. $s/n$
- **Now element $n+1$ arrives**
- **Inductive step:** For elements already in $S$, probability that the algorithm keeps it in $S$ is:
  \[
  \left(1 - \frac{s}{n+1}\right) + \left(\frac{s}{n+1}\right)\left(\frac{s - 1}{s}\right) = \frac{n}{n+1}
  \]
  - Element $n+1$ discarded
  - Element $n+1$ not discarded
  - Element in the sample not picked
- So, at time $n$, tuples in $S$ were there with prob. $s/n$
- Time $n\to n+1$, tuple stayed in $S$ with prob. $n/(n+1)$
- So prob. tuple is in $S$ at time $n+1 = \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$
Queries over a (long) Sliding Window
A useful model of stream processing is that queries are about a *window* of length $N$ – the $N$ most recent elements received

**Interesting case:** $N$ is so large that the data cannot be stored in memory, or even on disk
- Or, there are so many streams that windows for all cannot be stored

**Amazon example:**
- For every product $X$ we keep 0/1 stream of whether that product was sold in the $n$-th transaction
- We want answer queries, how many times have we sold $X$ in the last $k$ sales
Sliding Window: 1 Stream

- Sliding window on a single stream: \( N = 6 \)

```
N = 6
```

```
q w e r t y u i o p a s d f g h j k l z x c v b n m
```

```
q w e r t y u i o p a s d f g h j k l z x c v b n m
```

```
q w e r t y u i o p a s d f g h j k l z x c v b n m
```

```
q w e r t y u i o p a s d f g h j k l z x c v b n m
```

```
q w e r t y u i o p a s d f g h j k l z x c v b n m
```

Past --------- \[ \text{Future} \]
Problem:
- Given a stream of 0s and 1s
- Be prepared to answer queries of the form How many 1s are in the last \( k \) bits? where \( k \leq N \)

Obvious solution:
- Store the most recent \( N \) bits
- When new bit comes in, discard the \( N+1 \)st bit

Suppose \( N=6 \):
You can not get an exact answer without storing the entire window

Real Problem:
What if we cannot afford to store $N$ bits?
- E.g., we’re processing 1 billion streams and $N = 1$ billion

But we are happy with an approximate answer
An attempt: Simple solution

- **Q:** How many 1s are in the last \( N \) bits?
- A simple solution that does not really solve our problem: **Uniformity assumption**

Maintain 2 counters:
- \( S \): number of 1s from the beginning of the stream
- \( Z \): number of 0s from the beginning of the stream

How many 1s are in the last \( N \) bits? \( N \cdot \frac{S}{S+Z} \)

But, what if stream is non-uniform?
- What if distribution changes over time?
DGIM Method

- DGIM solution that does not assume uniformity
- We store $O(\log^2 N)$ bits per stream
- Solution gives approximate answer, never off by more than 50%
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits
Idea: Exponential Windows

- **Solution that doesn’t (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region

We can reconstruct the count of the last $N$ bits, except we are not sure how many of the last 6 1s are included in the $N$.
What’s Good?

- Stores only $O(\log^2 N)$ bits
  - $O(\log N)$ counts of $\log_2 N$ bits each
- Easy update as more bits enter
- Error in count no greater than the number of 1s in the “unknown” area
What’s Not So Good?

- As long as the 1s are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the 1s are in the unknown area at the end
- In that case, **the error is unbounded!**

Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of 1s:
  - Let the block *sizes* (number of 1s) increase exponentially

- When there are few 1s in the window, block sizes stay small, so errors are small
DGIM: Timestamps

- Each bit in the stream has a timestamp, starting $1, 2, \ldots$

- Record timestamps modulo $N$ (the window size), so we can represent any relevant timestamp in $O(\log_2 N)$ bits
DGIM: Buckets

- A *bucket* in the DGIM method is a record consisting of:
  - *(A)* The timestamp of its end \( [O(\log N) \text{ bits}] \)
  - *(B)* The number of 1s between its beginning and end \( [O(\log \log N) \text{ bits}] \)

- **Constraint on buckets:**
  - Number of 1s must be a power of 2
  - That explains the \( O(\log \log N) \) in *(B)* above

Representing a Stream by Buckets

- Either one or two buckets with the same power-of-2 number of 1s
- Buckets do not overlap in timestamps
- Buckets are sorted by size
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is \( > N \) time units in the past
Example: Bucketized Stream

Three properties of buckets that are maintained:
- Either one or two buckets with the same power-of-2 number of 1s
- Buckets do not overlap in timestamps
- Buckets are sorted by size

At least 1 of size 16. Partially beyond window.

2 of size 8

2 of size 4

1 of size 2

2 of size 1

100101011000101101010101010101101010101010101101010111010100010110010
When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to \( N \) time units before the current time.

- **2 cases:** Current bit is 0 or 1.
- **If the current bit is 0:** no other changes are needed.
If the current bit is 1:

1. Create a new bucket of size 1, for just this bit
   - End timestamp = current time
2. If there are now three buckets of size 1, combine the oldest two into a bucket of size 2
3. If there are now three buckets of size 2, combine the oldest two into a bucket of size 4
4. And so on ...
Example: Updating Buckets

Current state of the stream:

```
10010111000101101010101101010001011110101010101010101
```

Bit of value 1 arrives

```
001010111000101101010101010111010101010101000010110101
```

Two orange buckets get merged into a yellow bucket

```
00101011100010110101010101011101010101010100010110101
```

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

```
01011000101101010101010101011101010101010100010110101
```

Buckets get merged...

```
01011000101101010101010101011101010101010100010110101
```

State of the buckets after merging

```
01011000101101010101010101011101010101010100010110101
```
To estimate the number of 1s in the most recent $N$ bits:

1. Sum the sizes of all buckets but the last
   (note “size” means the number of 1s in the bucket)
2. Add half the size of the last bucket

**Remember:** We do not know how many 1s of the last bucket are still within the wanted window
Example: Bucketized Stream

At least 1 of size 16. Partially beyond window.

2 of size 8

2 of size 4

1 of size 2

2 of size 1

N
**Error Bound: Proof**

- **Why is error 50%? Let’s prove it!**
- Suppose the last bucket has size $2^r$
- Then by assuming $2^{r-1}$ (i.e., half) of its 1s are still within the window, we make an error of at most $2^{r-1}$
- Since there is at least one bucket of each of the sizes less than $2^r$, the true sum is at least $1 + 2 + 4 + .. + 2^{r-1} = 2^r - 1$
- Thus, error at most **50%**
Further Reducing the Error

- Instead of maintaining 1 or 2 of each size bucket, we allow either \( r-1 \) or \( r \) buckets \((r > 2)\)
  - Except for the largest size buckets; we can have any number between 1 and \( r \) of those
- Error is at most \( O(1/r) \)
- By picking \( r \) appropriately, we can tradeoff between number of bits we store and the error
Extensions

- Can we use the same trick to answer queries
  How many 1’s in the last $k$? where $k < N$?
  - A: Find earliest bucket $B$ that at overlaps with $k$.
    Number of 1s is the sum of sizes of more recent buckets + $\frac{1}{2}$ size of $B$

Can we handle the case where the stream is not bits, but integers, and we want the sum of the last $k$ elements?
Stream of positive integers

We want the sum of the last $k$ elements

- **Amazon**: Avg. price of last $k$ sales

**Solution:**

1. If you know all have at most $m$ bits
   - Treat $m$ bits of each integer as a separate stream
   - Use DGIM to count 1s in each integer
   - The sum is $\sum_{i=0}^{m-1} c_i 2^i$

2. Use buckets to keep partial sums
   - Sum of elements in size $b$ bucket is at most $2^b$

**Idea:** Sum in each bucket is at most $2^b$ (unless bucket has only 1 integer)

**Bucket sizes:**

<table>
<thead>
<tr>
<th>Bucket Size</th>
<th>Elements</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
Summary

- Sampling a fixed proportion of a stream
  - Sample size grows as the stream grows
- Sampling a fixed-size sample
  - Reservoir sampling
- Counting the number of 1s in the last N elements
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements