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Mining Data Streams (Part 2)

Mining of Massive Datasets Jure Leskovec, Anand Rajaraman, Jeff Ullman Stanford University http://www.mmds.org



Today's Lecture

More algorithms for streams:

• (1) Filtering a data stream: Bloom filters

Select elements with property x from stream

(2) Counting distinct elements: Flajolet-Martin

 Number of distinct elements in the last k elements of the stream

(3) Estimating moments: AMS method

- Estimate std. dev. of last *k* elements
- (4) Counting frequent items

(1) Filtering Data Streams

Filtering Data Streams

- Each element of data stream is a tuple
- Given a list of keys S
- Determine which tuples of stream are in S
- Obvious solution: Hash table
 - But suppose we do not have enough memory to store all of S in a hash table
 - E.g., we might be processing millions of filters on the same stream

Applications

Example: Email spam filtering

- We know 1 billion "good" email addresses
- If an email comes from one of these, it is NOT spam

Publish-subscribe systems

- You are collecting lots of messages (news articles)
- People express interest in certain sets of keywords
- Determine whether each message matches user's interest

First Cut Solution (1)

- Given a set of keys S that we want to filter
- Create a bit array B of n bits, initially all Os
- Choose a hash function h with range [0,n]
- Hash each member of s f S to one of n buckets, and set that bit to 1, i.e., B[h(s)]=1
- Hash each element *a* of the stream and output only those that hash to bit that was set to 1
 - Output a if B[h(a)] == 1

First Cut Solution (2)



Output the item since it may be in *S*. Item hashes to a bucket that at least one of the items in *S* hashed to.

Drop the item. It hashes to a bucket set to **0** so it is surely not in *S*.

Creates false positives but no false negatives

If the item is in S we surely output it, if not we may still output it

First Cut Solution (3)

- |S| = 1 billion email addresses
 |B| = 1GB = 8 billion bits
- If the email address is in *S*, then it surely hashes to a bucket that has the big set to 1, so it always gets through (*no false negatives*)
- Approximately 1/8 of the bits are set to 1, so about 1/8th of the addresses not in S get through to the output (*false positives*)
 - Actually, less than 1/8th, because more than one address might hash to the same bit

<u>Analysis:</u> Throwing Darts (1)

- More accurate analysis for the number of false positives
- Consider: If we throw *m* darts into *n* equally likely targets, what is the probability that a target gets at least one dart?

In our case:

- Targets = bits/buckets
- Darts = hash values of items

<u>Analysis:</u> Throwing Darts (2)

We have *m* darts, *n* targets

What is the probability that a target gets at least one dart?



Analysis: Throwing Darts (3)

- Fraction of 1s in the array B =
 = probability of false positive = 1 e^{-m/n}
- Example: 10⁹ darts, 8·10⁹ targets
 - Fraction of 1s in B = 1 e^{-1/8} = 0.1175
 - Compare with our earlier estimate: 1/8 = 0.125

Bloom Filter

- Consider: |S| = m, |B| = n
- Use k independent hash functions h₁,..., h_k
- Initialization:
 - Set B to all Os
 - Hash each element s ∈ S using each hash function h_i, set B[h_i(s)] = 1 (for each i = 1,.., k) (note: we have a single array B!)

Run-time:

- When a stream element with key x arrives
 - If $B[h_i(x)] = 1$ for all i = 1, ..., k then declare that x is in S
 - That is, x hashes to a bucket set to 1 for every hash function h_i(x)
 - Otherwise discard the element x

Bloom Filter -- Analysis

What fraction of the bit vector B are 1s?

- Throwing k·m darts at n targets
- So fraction of 1s is (1 e^{-km/n})
- But we have k independent hash functions and we only let the element x through if all k hash element x to a bucket of value 1
- So, false positive probability = (1 e^{-km/n})^k

Bloom Filter – Analysis (2)

- m = 1 billion, n = 8 billion
 - **k** = 1: (1 − e^{-1/8}) = **0.1175**
 - **k = 2**: (1 − e^{-1/4})² = **0.0493**
- What happens as we keep increasing k?



- "Optimal" value of k: n/m ln(2)
 - In our case: Optimal k = 8 ln(2) = 5.54 ≈ 6
 - Error at k = 6: $(1 e^{-1/6})^2 = 0.0235$

Bloom Filter: Wrap-up

- Bloom filters guarantee no false negatives, and use limited memory
 - Great for pre-processing before more expensive checks
- Suitable for hardware implementation
 - Hash function computations can be parallelized
- Is it better to have 1 big B or k small Bs?
 - It is the same: (1 e^{-km/n})^k vs. (1 e^{-m/(n/k)})^k
 - But keeping 1 big B is simpler

(2) Counting Distinct Elements

Counting Distinct Elements

Problem:

- Data stream consists of a universe of elements chosen from a set of size N
- Maintain a count of the number of distinct elements seen so far
- Obvious approach:

Maintain the set of elements seen so far

 That is, keep a hash table of all the distinct elements seen so far

Applications

- How many different words are found among the Web pages being crawled at a site?
 - Unusually low or high numbers could indicate artificial pages (spam?)
- How many different Web pages does each customer request in a week?
- How many distinct products have we sold in the last week?

Using Small Storage

- Real problem: What if we do not have space to maintain the set of elements seen so far?
- Estimate the count in an unbiased way
- Accept that the count may have a little error, but limit the probability that the error is large

Flajolet-Martin Approach

- Pick a hash function *h* that maps each of the
 N elements to at least log₂ *N* bits
- For each stream element *a*, let *r(a)* be the number of trailing **0s** in *h(a)*
 - r(a) = position of first 1 counting from the right
 - E.g., say h(a) = 12, then 12 is 1100 in binary, so r(a) = 2
- Record R = the maximum r(a) seen
 - R = max_a r(a), over all the items a seen so far

Estimated number of distinct elements = 2^R

Why It Works: Intuition

- <u>Very very rough and heuristic</u> intuition why Flajolet-Martin works:
 - h(a) hashes a with equal prob. to any of N values
 - Then h(a) is a sequence of log₂ N bits, where 2^{-r} fraction of all as have a tail of r zeros
 - About 50% of *a*s hash to ***0
 - About 25% of *a*s hash to ****00**
 - So, if we saw the longest tail of *r=2* (i.e., item hash ending *100) then we have probably seen about 4 distinct items so far
 - So, it takes to hash about 2^r items before we see one with zero-suffix of length r

Why It Works: More formally

- Now we show why Flajolet-Martin works
- Formally, we will show that probability of finding a tail of r zeros:
 - Goes to 1 if $m \gg 2^r$
 - Goes to 0 if $m \ll 2^r$

where m is the number of distinct elements seen so far in the stream

Thus, 2^R will almost always be around m!

Why It Works: More formally

- What is the probability that a given h(a) ends in at least r zeros is 2^{-r}
 - h(a) hashes elements uniformly at random
 - Probability that a random number ends in at least *r* zeros is 2^{-r}
- Then, the probability of NOT seeing a tail of length r among m elements:



Why It Works: More formally

• Note:
$$(1-2^{-r})^m = (1-2^{-r})^{2^r(m2^{-r})} \approx e^{-m2^{-r}}$$

Prob. of NOT finding a tail of length r is:

If *m* << 2^r, then prob. tends to 1

•
$$(1-2^{-r})^m \approx e^{-m2^{-r}} = 1$$
 as $m/2^r \rightarrow 0$

So, the probability of finding a tail of length r tends to 0

- If *m* >> 2^r, then prob. tends to 0
 - $(1-2^{-r})^m \approx e^{-m2^{-r}} = 0$ as **m/2**^r→∞

So, the probability of finding a tail of length r tends to 1

Thus, 2^R will almost always be around m!

Why It Doesn't Work

E[2^R] is actually infinite

- Probability halves when $R \rightarrow R+1$, but value doubles
- Workaround involves using many hash functions h_i and getting many samples of R_i
- How are samples R_i combined?
 - Average? What if one very large value 2^Ri?
 - Median? All estimates are a power of 2
 - Solution:
 - Partition your samples into small groups
 - Take the median of groups
 - Then take the average of the medians

(3) Computing Moments

Generalization: Moments

- Suppose a stream has elements chosen from a set A of N values
- Let m_i be the number of times value i occurs in the stream
- The kth moment is

$$\sum_{i\in A} (m_i)^k$$



 $\sum_{i\in A} (m_i)^k$

- Othmoment = number of distinct elements
 - The problem just considered
- 1st moment = count of the numbers of elements = length of the stream
 - Easy to compute
- 2nd moment = surprise number S =

a measure of how uneven the distribution is

Example: Surprise Number

- Stream of length 100
- 11 distinct values
- Item counts: 90, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
 Surprise S = 8,110

AMS Method

- AMS method works for all moments
- Gives an unbiased estimate
- We will just concentrate on the 2nd moment S
- We pick and keep track of many variables X:
 - For each variable X we store X.el and X.val
 - X.el corresponds to the item i
 - X.val corresponds to the count of item i
 - Note this requires a count in main memory, so number of X is limited
- Our goal is to compute $S = \sum_i m_i^2$

One Random Variable (X)

How to set X.val and X.el?

- Assume stream has length *n* (we relax this later)
- Pick some random time *t* (*t<n*) to start, so that any time is equally likely
- Let at time t the stream have item i. We set X.el = i
- Then we maintain count *c* (*X.val* = *c*) of the number of *is* in the stream starting from the chosen time *t*

• Then the estimate of the 2nd moment ($\sum_i m_i^2$) is: $S = f(X) = n (2 \cdot c - 1)$

• Note, we will keep track of multiple Xs, $(X_1, X_2, ..., X_k)$ and our final estimate will be $S = 1/k \sum_{j=1}^{k} f(X_j)$

Expectation Analysis



- 2nd moment is $S = \sum_i m_i^2$
- *c_t* ... number of times item at time *t* appears from time *t* onwards (*c₁=m_a*, *c₂=m_a-1*, *c₃=m_b*)

$$E[f(X)] = \frac{1}{n} \sum_{t=1}^{n} n(2c_t - 1)$$

$$= \frac{1}{n} \sum_{i=1}^{n} n(1 + 3 + 5 + \dots + 2m_i - 1)$$

$$m_i \dots \text{ total count of item i in the stream (we are assuming stream has length n)}$$

Group times by the value seen Time t when the last *i* is seen (*c*,=1) Time **t** when the penultimate **i** is seen (**c**_t=**2**) Time **t** when the first **i** is seen (**c**_t=**m**_j)

Expectation Analysis



- So, $E[f(X)] = \sum_{i} (m_{i})^{2} = S$
- We have the second moment (in expectation)!

Higher-Order Moments

- For estimating kth moment we essentially use the same algorithm but change the estimate:
 - For **k=2** we used *n* (2·c − 1)

For k=3 we use: n (3·c² – 3c + 1) (where c=X.val)

Why?

- For k=2: Remember we had (1 + 3 + 5 + ··· + 2m_i 1) and we showed terms 2c-1 (for c=1,...,m) sum to m²
 - $\sum_{c=1}^{m} 2c 1 = \sum_{c=1}^{m} c^2 \sum_{c=1}^{m} (c 1)^2 = m^2$
 - So: $2c 1 = c^2 (c 1)^2$
- For k=3: c³ (c-1)³ = 3c² 3c + 1

• Generally: Estimate = $n (c^k - (c - 1)^k)$

Combining Samples

In practice:

- Compute f(X) = n(2 c 1) for as many variables X as you can fit in memory
- Average them in groups
- Take median of averages

Problem: Streams never end

- We assumed there was a number *n*, the number of positions in the stream
- But real streams go on forever, so n is a variable – the number of inputs seen so far

Streams Never End: Fixups

- (1) The variables X have n as a factor –
 keep n separately; just hold the count in X
- (2) Suppose we can only store k counts.
 We must throw some X out as time goes on:
 - Objective: Each starting time t is selected with probability k/n
 - Solution: (fixed-size sampling!)
 - Choose the first k times for k variables
 - When the nth element arrives (n > k), choose it with probability k/n
 - If you choose it, throw one of the previously stored variables X out, with equal probability

Counting Itemsets

Counting Itemsets

- New Problem: Given a stream, which items appear more than s times in the window?
- Possible solution: Think of the stream of baskets as one binary stream per item
 - 1 = item present; 0 = not present
 - Use DGIM to estimate counts of 1s for all items



Extensions

- In principle, you could count frequent pairs or even larger sets the same way
 - One stream per itemset
- Drawbacks:
 - Only approximate
 - Number of itemsets is way too big

Exponentially Decaying Windows

- Exponentially decaying windows: A heuristic for selecting likely frequent item(sets)
 - What are "currently" most popular movies?
 - Instead of computing the raw count in last N elements
 - Compute a smooth aggregation over the whole stream
- If stream is $a_1, a_2, ...$ and we are taking the sum of the stream, take the answer at time t to be: $= \sum_{i=1}^{t} a_i (1-c)^{t-i}$

c is a constant, presumably tiny, like 10⁻⁶ or 10⁻⁹

 When new a_{t+1} arrives: Multiply current sum by (1-c) and add a_{t+1}

Example: Counting Items

- If each *a_i* is an "item" we can compute the characteristic function of each possible item *x* as an Exponentially Decaying Window
 - That is: $\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$ where $\delta_i = 1$ if $a_i = x$, and 0 otherwise
 - Imagine that for each item x we have a binary stream (1 if x appears, 0 if x does not appear)
 - New item x arrives:
 - Multiply all counts by (1-c)
 - Add +1 to count for element x
- Call this sum the "weight" of item x

Sliding Versus Decaying Windows



• Important property: Sum over all weights $\sum_t (1-c)^t$ is 1/[1-(1-c)] = 1/c

Example: Counting Items

- What are "currently" most popular movies?
- Suppose we want to find movies of weight > ½
 - Important property: Sum over all weights $\sum_t (1-c)^t$ is 1/[1-(1-c)] = 1/c
- Thus:
 - There cannot be more than 2/c movies with weight of ½ or more
- So, 2/c is a limit on the number of movies being counted at any time

Extension to Itemsets

Count (some) itemsets in an E.D.W.

- What are currently "hot" itemsets?
 - Problem: Too many itemsets to keep counts of all of them in memory

When a basket B comes in:

- Multiply all counts by (1-c)
- For uncounted items in B, create new count
- Add 1 to count of any item in B and to any itemset contained in B that is already being counted
- Drop counts < ½</p>
- Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset S ⊆ B if every proper subset of S had a count prior to arrival of basket B
 - Intuitively: If all subsets of S are being counted this means they are "frequent/hot" and thus S has a potential to be "hot"

Example:

- Start counting S={i, j} iff both i and j were counted prior to seeing B
- Start counting S={i, j, k} iff {i, j}, {i, k}, and {j, k} were all counted prior to seeing B

How many counts do we need?

- Counts for single items < (2/c)·(avg. number of items in a basket)
- Counts for larger itemsets = ??
- But we are conservative about starting counts of large sets
 - If we counted every set we saw, one basket of 20 items would initiate 1M counts